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International Journal of Thermal Sciences 46 (2007) 551–560
International Journal of Thermal Sciences
www.elsevier.com/locate/ijts

Forced convection with laminar pulsating flow in a channel or tube

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Received 15 February 2006; received in revised form 26 July 2006; accepted 27 July 2006

Available online 24 August 2006

Abstract

A perturbation approach is used to obtain analytical expressions for the velocity, temperature distribution, and transient Nusselt number for the problem of forced convection, in a parallel-plates channel or a circular tube, produced by an applied pressure gradient that fluctuates with small amplitude harmonically in time about a non-zero mean. It is found that the fluctuating part of this Nusselt number alters in magnitude and phase as the dimensionless frequency increases. The magnitude increases from zero, goes through a peak, and then decreases to zero. The height of the peak decreases as the Prandtl number Pr increases. The phase (relative to that of the steady component) decreases from $\pi/2$ to $-\pi/2$. When $Pr = 1$ there is a weak singularity in the form of the temperature distribution, but the Nusselt number is not significantly different from the case when $Pr \neq 1$.

Keywords: Forced convection; Pulsating flow; Weak singularity

1. Introduction

Forced convection with steady laminar flow in a parallel-plates channel or a circular tube is a classical problem, and is the subject of hundreds of papers. However, our literature survey revealed only a score of papers on the problem where the applied pressure gradient, and hence the velocity, is pulsating harmonically about a non-zero mean. Experimental work has been reported in [1–4] and numerical solutions have been presented in [5–13].

It appears that the first analytic studies of laminar pulsating forced convection in confined spaces were made by Siegel [14] (for a circular duct) and Siegel and Perlmutter [15] (for a parallel-plates channel). In each case the fluid velocity was approximated by slug flow. The work of Faghri et al. [16] for flow in a circular pipe treated the interaction between the velocity and temperature oscillations via an extra term in the energy equations, while Faghri et al. [17] used a finite-difference method to treat thermally developing convection in a parallel-plates channel. Moschandreou and Zamir [18] studied pulsating flow in a circular tube with uniform constant heat flux on the boundaries. They obtained an expression for fluctuating part of the temperature using a Green's function method. However, Hemida et al. [19] pointed out that the solution of Moschandreou and Zamir [18] did not satisfy the appropriate differential equation expressing the conservation of thermal energy. Hemida et al. [19] also used a Green's function method to find an analytical expression for the temperature distribution. In order to obtain the Nusselt number they resorted to a numerical calculation. Yu et al. [20] also considered the case of a circular tube with constant heat flux. They obtained a series expression for the transient temperature distribution. They did not present an analytical solution for the bulk temperature nor for the transient Nusselt number.

In the present paper a perturbation approach is followed. The case of the uniform heat-flux boundary condition is considered, and the parallel-plates channel and circular tube are considered in turn. The Péclet number is assumed to be large so that the axial heat conduction can be neglected. Analytical expressions for the velocity and temperature are obtained without further approximation. However, for the determination of the bulk temperature and the Nusselt number it is assumed that

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doi:10.1016/j.ijthermalsci.2006.07.011

Nomenclature

$G[1+\varepsilon\exp(i\Omega t^*)]$ applied pressure gradient
H half of the channel width
Nu Nusselt number, $\frac{2Hq''}{k(T_w^*-T_m^*)}$ for a channel and $\frac{2Rq''}{k(T_w^*-T_m^*)}$ for a tube
Pr Prandtl number, $\frac{\mu/\rho}{k_m/\rho c_P}$
q'' boundary heat flux
r dimensionless radial coordinate, r^*/R
r^* radial coordinate
R tube radius
t dimensionless time, $\mu t^*/\rho H^2$ for a channel and $\mu t^*/\rho R^2$ for a tube
t^* time
T^* temperature
T_m^* bulk temperature
T_w^* wall temperature
$\hat{T}$ dimensionless temperature, $\frac{T^*-T_w^*}{T_m^*-T_w^*}$
u dimensionless longitudinal velocity, $\mu u^*/GH^2$ for a channel and $\mu u^*/GR^2$ for a tube
u^* longitudinal velocity
$\bar{u}$ spatial average of u
$\bar{u}^*$ spatial average of u^*
$\hat{u}$ rescaled dimensionless longitudinal velocity, $u/\bar{u}$
x^* longitudinal coordinate
y^* transverse coordinate
y dimensionless transverse coordinate, y^*/H

Greek symbols

β parameter defined by Eq. (22), $(i\,Pr\,\omega)^{1/2}$
γ parameter defined by Eq. (7), $(i\omega)^{1/2}$
ω dimensionless frequency, which may be regarded as a Reynolds number based on $H\Omega$ as the velocity scale and H as the length scale, $\rho H^2\Omega/\mu$ for a channel and $\rho R^2\Omega/\mu$ for a tube

Superscripts

$*$ dimensional variable

the amplitude ε of the fluctuations is small, and expressions for these quantities are obtained to first order in ε.

2. Analysis: parallel-plates channel

We consider a parallel-plates channel with fluid velocity u^* in the x^*-direction, with the plates at $y^* = \pm H$. The asterisks denote dimensional variables. We suppose that the applied pressure gradient is $G[1+\varepsilon\exp(i\Omega t^*)]$, where t^* is the time. We assume that the fluid is incompressible, so that for this unidirectional flow the momentum equation can be written as

$$\rho\frac{\partial u^*}{\partial t^*} = G\left[1+\varepsilon\exp(i\Omega t^*)\right] + \mu\frac{\partial^2 u^*}{\partial y^{*2}} \tag{1}$$

Here ρ is the fluid density and μ is the fluid viscosity.

We let

$$y = \frac{y^*}{H}, \qquad u = \frac{\mu u^*}{GH^2}, \qquad t = \frac{\mu t^*}{\rho H^2}, \qquad \omega = \frac{\rho H^2\Omega}{\mu} \tag{2}$$

We note that the dimensionless frequency ω takes the form of a Reynolds number based on $H\Omega$ as the velocity scale and H as the length scale. The dimensionless parameter $\omega^{1/2}$ is sometimes called the Womersley parameter and $(\omega/2)^{1/2}$ has been called the Stokes parameter.

Then

$$\frac{\partial u}{\partial t} = 1 + \varepsilon\exp(i\omega t) + \frac{\partial^2 u}{\partial y^2} \tag{3}$$

This has to be solved subject to the boundary conditions

$$u = 0 \quad \text{at } y = \pm 1. \tag{4}$$

We denote the spatial average of u by $\bar{u}$ and introduce a new dimensionless velocity $\hat{u} = u/\bar{u}$ and write

$$\hat{u} = \hat{u}_0(y) + \varepsilon\hat{u}_1(y)e^{i\omega t} \tag{5}$$

It should be noted that we permit $\hat{u}_1(y)/\hat{u}_0(y)$ to be a complex quantity, in recognition of the existence of a phase difference between the velocity and the applied pressure gradient.

We substitute this in Eqs. (3) and (4) and equate the coefficients of the powers of ε. We also introduce the shorthand notations

$$F_\gamma = 1 - \frac{\cosh\gamma y}{\cosh\gamma} \tag{6a}$$

$$G_1 = 1 - y^2 \tag{6b}$$

$$H_1 = 5 - 6y^2 + y^4 \tag{6c}$$

$$a = \frac{3i}{\omega} \tag{6d}$$

$$b = \frac{9i}{2\omega}\left(1 - \frac{\tanh\gamma}{\gamma}\right) = \frac{3a}{2}\left(1 - \frac{\tanh\gamma}{\gamma}\right) \tag{6e}$$

where

$$\gamma = (i\omega)^{1/2} \tag{7}$$

The solution for the velocity is readily found to be

$$\hat{u}_0 = \frac{3}{2}G_1 \tag{8a}$$

$$\hat{u}_1 = -aF_\gamma + bG_1 \tag{8b}$$

This solution is exact, independent of the magnitude of ε.

For the case of uniform flux boundaries, the thermal energy equation takes the form

$$\frac{\partial T^*}{\partial t^*} + u^*\frac{\partial T^*}{\partial x^*} = \frac{k}{\rho c_P}\frac{\partial^2 T^*}{\partial y^{*2}} \tag{9}$$

where k is the thermal conductivity and c_P is the specific heat, and

$$\frac{\partial T^*}{\partial x^*} = \frac{q''}{\rho c_P H\bar{u}^*} \tag{10}$$

where $\bar{u}^*$ is the spatial average of u^* and q'' is the boundary heat flux.

The Nusselt number is defined as

$$Nu = \frac{2Hq''}{k(T_w^* - T_m^*)} \tag{11}$$

Here T_w^* is the wall temperature and T_m^* is the bulk temperature defined by

$$T_m^* = \frac{1}{\bar{u}H}\int_0^H u^* T^* \, dy^* \tag{12}$$

We define a dimensionless temperature

$$\hat{T} = \frac{T^* - T_w^*}{T_m^* - T_w^*} \tag{13}$$

(It should be noted that this form of nondimensionalization can be made regardless of whether the wall thermal boundary condition is one of uniform temperature or uniform heat flux. For an example of the uniform treatment of forced convection with both boundary conditions in turn see Section 4.9 of [21].)

We also introduce the Prandtl number

$$Pr = \frac{\mu/\rho}{k_m/\rho c_P} \tag{14}$$

We now assume that ε is small compared with unity.

Then Eq. (9) takes the form

$$\frac{\partial^2 \hat{T}}{\partial y^2} - Pr\frac{\partial \hat{T}}{\partial t} = -\frac{1}{2}Nu\,\hat{u} \tag{15}$$

This must be solved subject to the boundary conditions

$$\hat{T} = 0 \quad \text{at } y = 1, \qquad \frac{\partial \hat{T}}{\partial y} = 0 \quad \text{at } y = 0 \tag{16}$$

The solution is

$$\hat{T} = \hat{T}_0 + \varepsilon \hat{T}_1 e^{i\omega t} \tag{17}$$

where (on substituting in Eqs. (15) and (16) and equating powers of ε)

$$\frac{d^2\hat{T}_0}{dy^2} = -\frac{1}{2}Nu\,\hat{u}_0 \tag{18}$$

$$\frac{d^2\hat{T}_1}{dy^2} - i\omega\, Pr\, T_1 = -\frac{1}{2}Nu\,\hat{u}_1 \tag{19}$$

The solutions, for the case $Pr \neq 1$ so that $\gamma \neq \beta$, are

$$\hat{T}_0 = \frac{Nu}{16}H_1 \tag{20}$$

$$\hat{T}_1 = \frac{Nu}{2}\left\{\frac{a}{\gamma^2 - \beta^2}F_\gamma + \left[\frac{a\gamma^2}{\beta^2(\beta^2 - \gamma^2)} - \frac{2b}{\beta^4}\right]F_\beta + \frac{b}{\beta^2}G_1\right\} \tag{21}$$

Here

$$\beta = (i\,Pr\,\omega)^{1/2} \tag{22}$$

$$F_\beta = 1 - \frac{\cosh\beta y}{\cosh\beta} \tag{23}$$

(It is clear that the expression in Eq. (21) becomes singular when $\beta = \gamma$, that is when $Pr = 1$. See Eq. (39) for the solution for the case $Pr = 1$.)

The compatibility condition (an identity required for the consistency of Eqs. (12) and (13)) is

$$\langle \hat{u}\hat{T}\rangle = 1 \tag{24}$$

To first order, this gives

$$\langle \hat{u}_0\hat{T}_0\rangle + \big(\langle \hat{u}_0\hat{T}_1\rangle + \langle \hat{u}_1\hat{T}_0\rangle\big)\varepsilon e^{i\omega t} = 1 \tag{25}$$

Substitution of Eqs. (20) and (21) into (25) gives Nu.

As intermediate results we have

$$\langle \hat{u}_0\hat{T}_0\rangle = \frac{3Nu}{32}\langle G_1 H_1\rangle \tag{26}$$

$$\langle \hat{u}_0\hat{T}_1\rangle = \frac{3Nu}{4}\left\{\frac{a}{\gamma^2 - \beta^2}\langle F_\gamma G_1\rangle + \left[\frac{a\gamma^2}{\beta^2(\beta^2 - \gamma^2)} - \frac{2b}{\beta^4}\right]\langle F_\beta G_1\rangle + \frac{b}{\beta^2}\langle G_1^2\rangle\right\} \tag{27}$$

$$\langle \hat{u}_1\hat{T}_0\rangle = Nu\left\{-\frac{a}{16}\langle F_\gamma H_1\rangle + \frac{b}{16}\langle G_1 H_1\rangle\right\} \tag{28}$$

where

$$\langle G_1 H_1\rangle = \frac{272}{105} \tag{29a}$$

$$\langle F_\gamma G_1\rangle = \frac{2}{3} - \frac{2}{\gamma^2} + \frac{2\tanh\gamma}{\gamma^3} \tag{29b}$$

$$\langle F_\beta G_1\rangle = \frac{2}{3} - \frac{2}{\beta^2} + \frac{2\tanh\beta}{\beta^3} \tag{29c}$$

$$\langle G_1^2\rangle = \frac{8}{15} \tag{29d}$$

$$\langle F_\gamma H_1\rangle = \frac{16}{5} - \frac{8}{\gamma^2} + \frac{24}{\gamma^4} - \frac{24\tanh\gamma}{\gamma^5} \tag{29e}$$

Hence

$$\langle \hat{u}_0\hat{T}_0\rangle = \frac{17}{70}Nu \tag{30}$$

$$\langle \hat{u}_0\hat{T}_1\rangle = \frac{1}{2}a\,Nu\left\{\frac{1}{\gamma^2 - \beta^2}\left(1 - \frac{3}{\gamma^2} + \frac{3\tanh\gamma}{\gamma^3}\right) + \frac{6}{5\beta^2}\left(1 - \frac{\tanh\gamma}{\gamma}\right) + \left[\frac{\gamma^2}{\beta^2(\beta^2 - \gamma^2)} - \frac{3}{\beta^4}\left(1 - \frac{\tanh\gamma}{\gamma}\right)\right]\left(1 - \frac{3}{\beta^2} + \frac{3\tanh\beta}{\beta^3}\right)\right\} \tag{31}$$

$$\langle \hat{u}_1\hat{T}_0\rangle = \frac{a\,Nu}{70}\left\{3 - \frac{17\tanh\gamma}{\gamma} + \frac{35}{\gamma^2} - \frac{105}{\gamma^4} + \frac{105\tanh\gamma}{\gamma^5}\right\} \tag{32}$$

Finally we have

$$Nu = \frac{70}{17}\left[1 - \frac{210i}{17\omega}\big\{P(\beta,\gamma) + Q(\gamma)\big\}\varepsilon e^{i\omega t}\right] \tag{33}$$

where

$$P(\beta,\gamma)=\frac{1}{2}\left\{\frac{1}{\gamma^2-\beta^2}\left(1-\frac{3}{\gamma^2}+\frac{3\tanh\gamma}{\gamma^3}\right)\right.$$
$$+\left[\frac{\gamma^2}{\beta^2(\beta^2-\gamma^2)}-\frac{3}{\beta^4}\left(1-\frac{\tanh\gamma}{\gamma}\right)\right]$$
$$\times\left(1-\frac{3}{\beta^2}+\frac{3\tanh\beta}{\beta^3}\right)$$
$$\left.+\frac{6}{5\beta^2}\left(1-\frac{\tanh\gamma}{\gamma}\right)\right\} \tag{34}$$

$$Q(\gamma)=\frac{1}{70}\left[3-\frac{17\tanh\gamma}{\gamma}+\frac{35}{\gamma^2}-\frac{105}{\gamma^4}+\frac{105\tanh\gamma}{\gamma^5}\right] \tag{35}$$

and we recall that $\beta=(i\,Pr\,\omega)^{1/2}$ and $\gamma=(i\omega)^{1/2}$.

For the case of small values of ω, the asymptotic result is

$$Nu=\frac{70}{17}\left[1+\frac{8i\omega}{765}\varepsilon e^{i\omega t}\right] \tag{36}$$

It is remarkable that this result is independent of Pr, and that the terms $\langle\hat{u}_0\hat{T}_1\rangle$ and $\langle\hat{u}_1\hat{T}_0\rangle$ contribute equally at this level of approximation. The oscillatory component has phase $\pi/2$ ahead of that of the steady component. In obtaining Eq. (36) is necessary to retain five non-zero terms in the series expansions for $P(\beta,\gamma)$ and $Q(\gamma)$, which are formally singular at $\omega=0$.

For the case of large values of ω, the asymptotic result is

$$Nu=\frac{70}{17}\left[1-\frac{9i}{17\omega}\varepsilon e^{i\omega t}\right] \tag{37}$$

This expression is also independent of Pr, and now $\langle\hat{u}_0\hat{T}_1\rangle$ does not contribute at this level of approximation. The oscillatory component has phase $\pi/2$ behind that of the steady component.

We present our results in terms of Nu_t/Nu_s, where

$$Nu=Nu_s+Nu_t\varepsilon e^{i\omega t} \tag{38}$$

For the special case $Pr=1$, the above solution is formally singular, but one can find the expressions for $\hat{T}_1$ and Nu either by solving Eq. (19) and the consequent equations from scratch, or by taking the limit as β tends to γ in Eqs. (21) and (34). By either route one finds that

$$\hat{T}_1=\frac{Nu}{2}\left\{-\left(\frac{a}{\gamma^2}+\frac{2b}{\gamma^4}\right)F_\gamma\right.$$
$$\left.+\frac{a}{2\gamma\cosh\gamma}(\tanh\gamma\cosh\gamma y-y\sinh\gamma y)+\frac{b}{\beta^2}G_1\right\} \tag{39}$$

$$\langle\hat{u}_0\hat{T}_1\rangle=a\,Nu\left\{\frac{1}{10\gamma^2}+\frac{9}{4\gamma^4}+\frac{9}{2\gamma^6}\right.$$
$$-\left(\frac{3}{5\gamma^3}+\frac{9}{4\gamma^5}+\frac{9}{\gamma^7}\right)\tanh\gamma$$
$$\left.-\left(\frac{3}{4\gamma^4}-\frac{9}{2\gamma^8}\right)\tanh^2\gamma\right\} \tag{40}$$

One observes that although Eq. (39) differs from Eq. (21) by the inclusion of a term proportional to y, the solution remains bounded because y remains bounded. Thus one does not have true resonance in this situation. The singularity is a weak one, and one finds that Nu for the case $Pr=1$ is well approximated numerically simply by inputting the value 0.999 or the value 1.001 for Pr.

3. Analysis: circular tube

The analysis for the case of a circular tube follows closely that for a parallel-plates channel, and so for brevity we just list the major changes. The tube radius R replaces H in the definitions of dimensionless parameters such as ω and Nu and y is replaced by the radial coordinate $r=r^*/R$.

The momentum equation

$$\frac{\partial u}{\partial t}=1+\varepsilon\exp(i\omega t)+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r} \tag{41}$$

has to be solved subject to the boundary conditions

$$u=0 \quad \text{at } r=\pm 1 \tag{42}$$

The normalized solution is

$$\hat{u}=\frac{u}{\bar{u}}=\hat{u}_0+\varepsilon\hat{u}_1e^{i\omega t} \tag{43}$$

In terms of the newly defined quantities

$$F_\gamma=1-\frac{I_0(\gamma r)}{I_0(\gamma)} \tag{44a}$$

$$G_2=1-r^2 \tag{44b}$$

$$H_2=3-4r^2+r^4 \tag{44c}$$

$$a=\frac{8i}{\omega} \tag{44d}$$

$$b=\frac{16i}{\omega}\left(1-\frac{2I_1(\gamma)}{\gamma I_0(\gamma)}\right)=2a\left(1-\frac{2I_1(\gamma)}{\gamma I_0(\gamma)}\right) \tag{44e}$$

$$\text{where } \gamma=(i\omega)^{1/2}, \tag{45}$$

one finds that

$$\hat{u}_0=2G_2 \tag{46a}$$

$$\hat{u}_1=-aF_\gamma+bG_2 \tag{46b}$$

The thermal energy equation now becomes

$$\frac{\partial^2\hat{T}}{\partial r^2}+\frac{1}{r}\frac{\partial\hat{T}}{\partial r}-\lambda\frac{\partial\hat{T}}{\partial t}=-Nu\,\hat{u} \tag{47}$$

This must solved subject to the boundary conditions

$$\hat{T}=0 \quad \text{at } r=1 \quad \text{and} \quad \hat{T} \text{ finite} \quad \text{at } r=0 \tag{48}$$

The solution is

$$\hat{T}=\hat{T}_0+\varepsilon\hat{T}_1e^{i\omega t} \tag{49}$$

where

$$\hat{T}_0=\frac{Nu}{8}H_2 \tag{50}$$

$$\hat{T}_1=Nu\left\{\frac{a}{\gamma^2-\beta^2}F_\gamma+\left[\frac{a\gamma^2}{\beta^2(\beta^2-\gamma^2)}-\frac{4b}{\beta^4}\right]F_\beta+\frac{b}{\beta^2}G_2\right\} \tag{51}$$

where

$$\beta=(i\omega Pr)^{1/2}, \qquad F_\beta=1-\frac{I_0(\beta r)}{I_0(\beta)} \tag{52}$$

The compatibility condition is still

$$\langle\hat{u}\hat{T}\rangle=1 \tag{53}$$

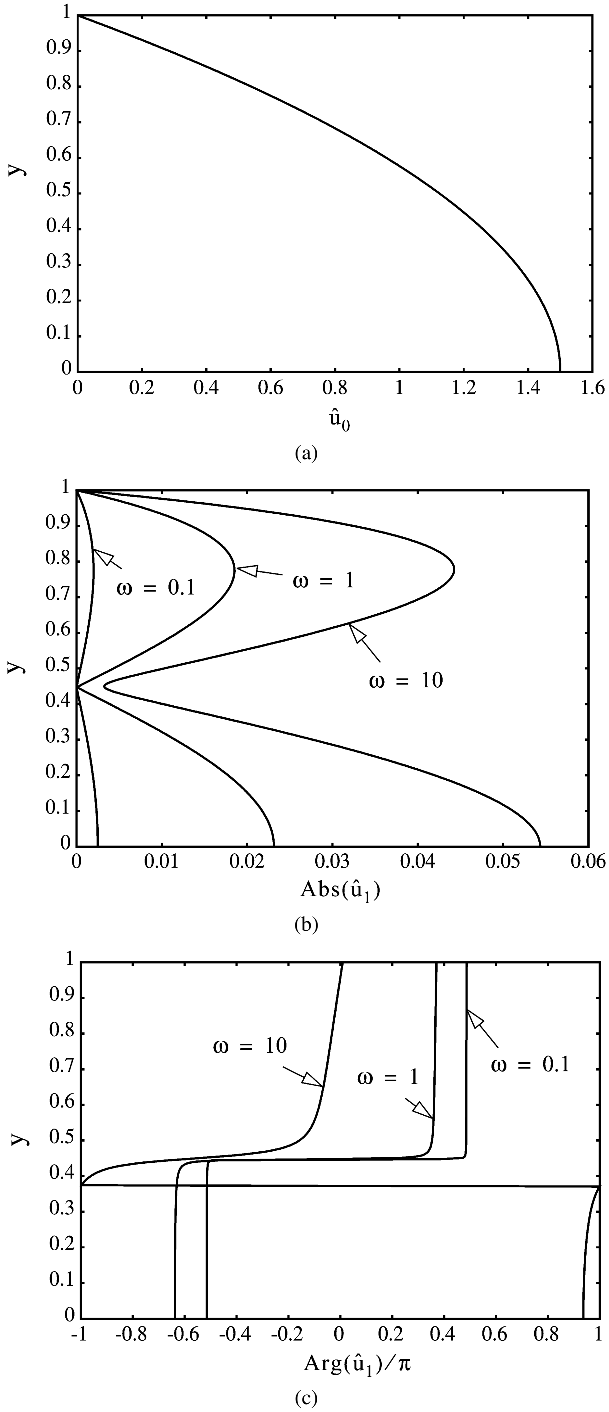


Fig. 1. Plots of the profile of (a) the normalized steady-state velocity, and (b) the modulus and (c) the argument of the normalized perturbation velocity, for various values of the dimensionless frequency, each for the parallel-plates channel.

but now

$$\langle f(r)\rangle = 2\int_0^1 f(r)r\,dr. \tag{54}$$

From these equations one concludes that

$$Nu = \frac{48}{11}\left[1 - \frac{384i\varepsilon}{11\omega}\left\{P(\beta,\gamma) + Q(\gamma)\right\}e^{i\omega t}\right] \tag{55}$$

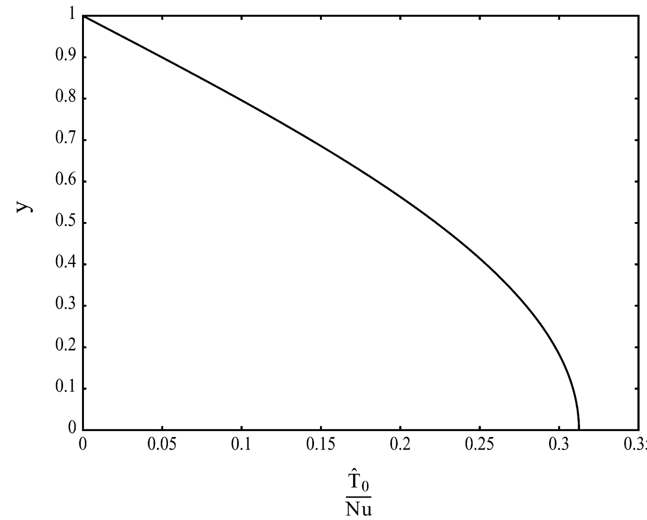


Fig. 2. Plot of the profile of the normalized steady-state temperature, for the parallel-plates channel.

where

$$\begin{aligned} P(\beta,\gamma) = {}& \frac{1}{\gamma^2-\beta^2}\left(1 - \frac{8}{\gamma^2} + \frac{16I_1(\gamma)}{\gamma^3 I_0(\gamma)}\right) \\ & + \frac{4}{3\beta^2}\left(1 - \frac{2I_1(\gamma)}{\gamma I_0(\gamma)}\right) \\ & + \left[\frac{\gamma^2}{\beta^2(\beta^2-\gamma^2)} - \frac{8}{\beta^4}\left(1 - \frac{2I_1(\gamma)}{\gamma I_0(\gamma)}\right)\right] \\ & \times \left(1 - \frac{8}{\beta^2} + \frac{16I_1(\beta)}{\beta^3 I_0(\beta)}\right) \end{aligned} \tag{56}$$

$$Q(\gamma) = -\frac{1}{6} + \frac{1}{\gamma^2} - \frac{8}{\gamma^4} + \frac{16I_1(\gamma)}{\gamma^5 I_0(\gamma)} + \frac{11}{48}\left(1 - \frac{2I_1(\gamma)}{\gamma I_0(\gamma)}\right) \tag{57}$$

For the case of small values of ω, the asymptotic result is

$$Nu = \frac{48}{11}\left[1 + \frac{2i\omega}{165}\varepsilon e^{i\omega t}\right] \tag{58}$$

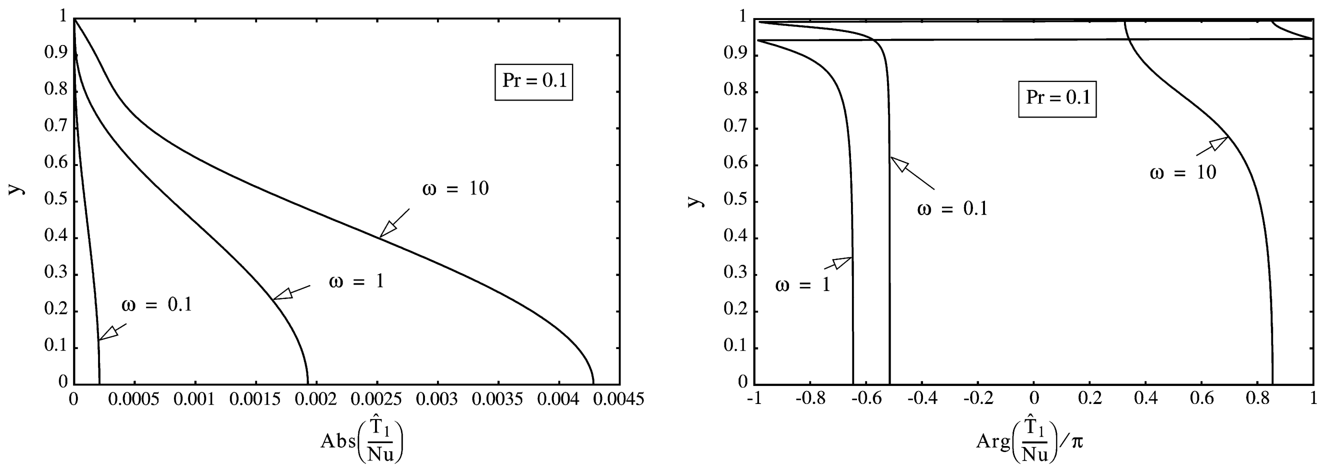
For the case of large values of ω, the asymptotic result is

$$Nu = \frac{48}{11}\left[1 - \frac{24i\varepsilon}{11\omega}e^{i\omega t}\right] \tag{59}$$

For the special case of $Pr = 1$ one has

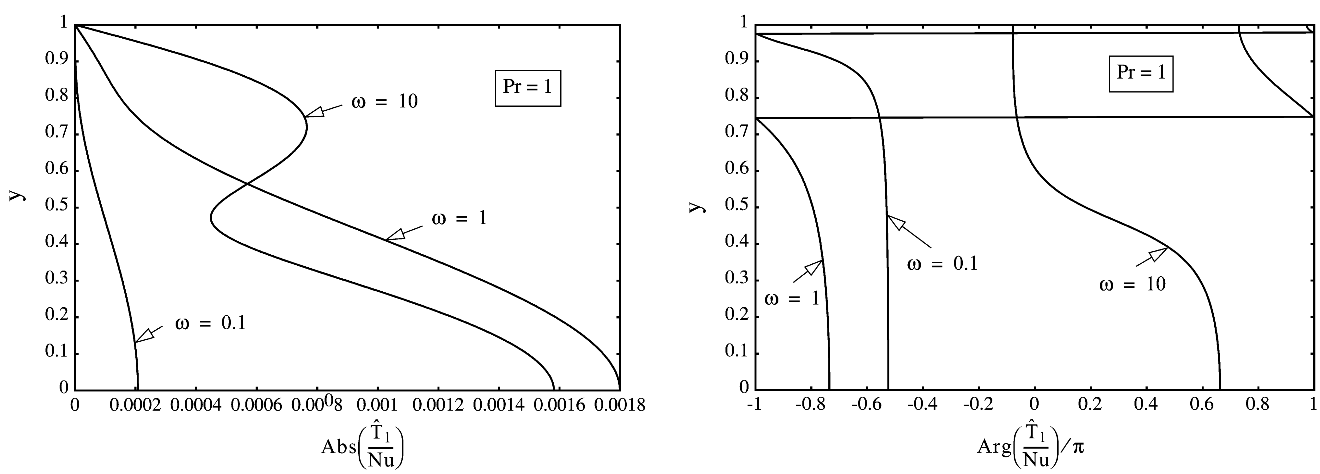
$$\begin{aligned} \hat{T}_1 = Nu\bigg\{ & -\left(\frac{a}{\gamma^2} + \frac{4b}{\gamma^4}\right)F_\gamma \\ & + \frac{a}{2\gamma[I_0(\gamma)]^2}\big(I_1(\gamma)I_0(\gamma r) - I_0(\gamma)rI_1(\gamma r)\big) \\ & + \frac{b}{\beta^2}G_2\bigg\} \end{aligned} \tag{60}$$

$$\begin{aligned} \langle \hat{u}_0\hat{T}_1\rangle = a\,Nu\bigg\{ & \frac{1}{3\gamma^2} + \frac{16}{\gamma^4} + \frac{64}{\gamma^6} - \left(\frac{8}{3\gamma^3} + \frac{32}{\gamma^5} + \frac{256}{\gamma^7}\right) \\ & \times \frac{I_1(\gamma)}{I_0(\gamma)} - \left(\frac{8}{\gamma^4} - \frac{256}{\gamma^8}\right)\left[\frac{I_1(\gamma)}{I_0(\gamma)}\right]^2\bigg\} \end{aligned} \tag{61}$$



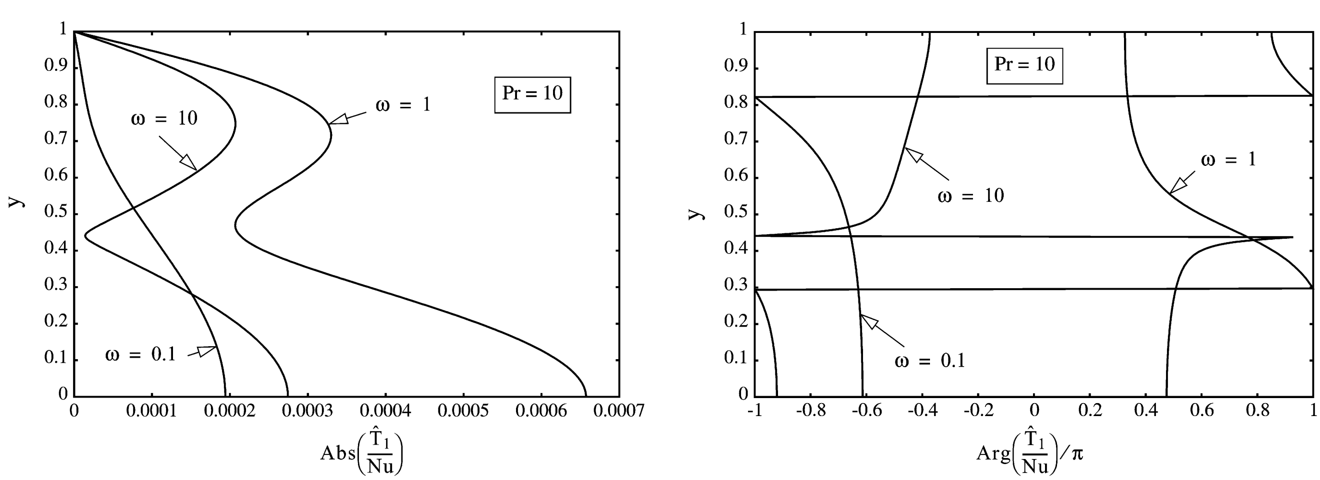
(a) (b)

Fig. 3. Plot of the profile of (a) the modulus, (b) the argument of the normalized perturbation temperature, for various values of the dimensionless frequency, for the parallel-plates channel. Prandtl number $Pr = 0.1$.



(a) (b)

Fig. 4. As for Fig. 4, but with $Pr = 1$.



(a) (b)

Fig. 5. As for Fig. 4, but with $Pr = 10$.

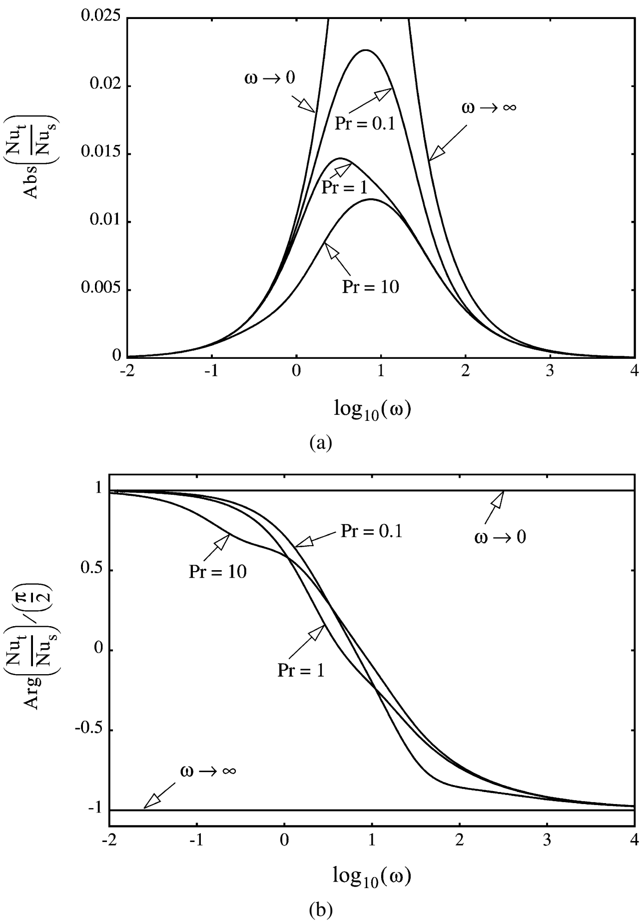


Fig. 6. Plots of the (a) the modulus, and (b) the argument as a fraction of $\pi/2$, of Nu_t/Nu_s, where $Nu = Nu_s + Nu_t\,\varepsilon e^{i\omega t}$, for various values of the Prandtl number Pr, for the case of a parallel-plates channel. Here $Nu_s = 70/17$.

4. Results and discussion

4.1. Parallel-plates channel

The steady-state velocity profile is presented in Fig. 1(a). This is the well known unimodal profile for plane Poiseuille flow. In Figs. 1(b) and 1(c) the perturbation velocity profile is presented. It is seen the central peak is flanked by a subsidiary peak. The amplitude of the peaks increases as the dimensionless frequency increases. Also, it is seen that in a central core of the channel the perturbation velocity has phase behind the applied pressure gradient, while in an outer sheath the perturbation velocity has phase ahead of the applied pressure gradient.

The steady-state temperature profile is presented in Fig. 2, while in Figs. 3–5 the perturbation temperature profiles are plotted. Fig. 3 shows that for a small value of the Prandtl number ($Pr = 0.1$) the situation is relatively simple; the plots have a single (central) peak whose amplitude increases as the frequency increases, and over most of the channel the phase (advanced for large ω and retarded for small and intermediate ω) does not vary much. For $Pr = 1$ (Fig. 4), for large values of ω there is a secondary peak and the phase jumps around. For a large value of the Prandtl number ($Pr = 10$, Fig. 5), the secondary peak and the phase jumps occur for intermediate as well as large values of ω.

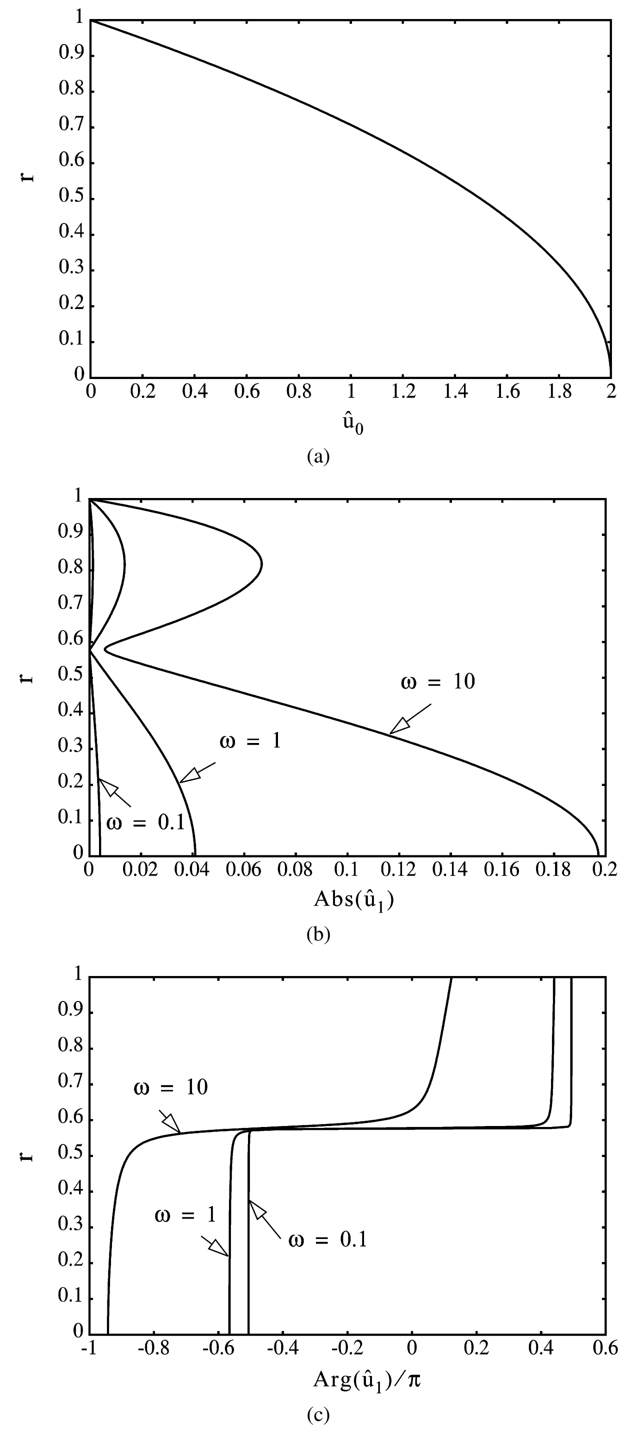


Fig. 7. Plots of the profile of (a) the normalized steady-state velocity, and (b) the modulus and (c) the argument of the normalized perturbation velocity, for various values of the dimensionless frequency, each for the circular tube.

In Fig. 6 the Nusselt number results are presented. It is now clear that the asymptotic expressions for small and large values of ω give upper bounds. The modulus of Nu_t/Nu_s peaks at an ω value of order 5, and the height of the peak decreases

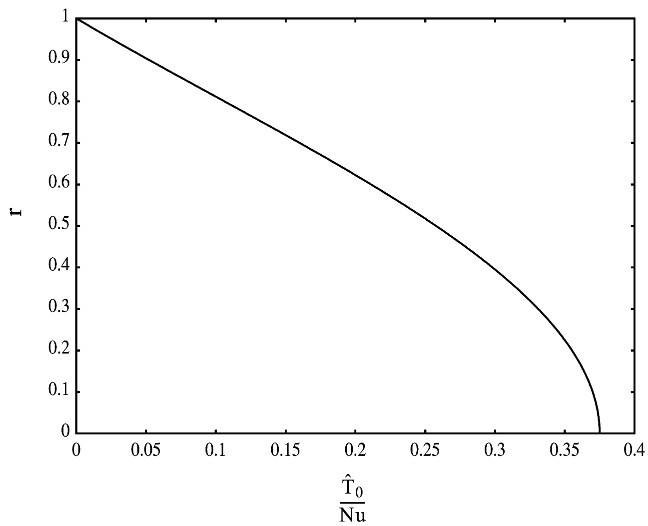


Fig. 8. Plot of the profile of the normalized steady-state temperature, for the circular tube.

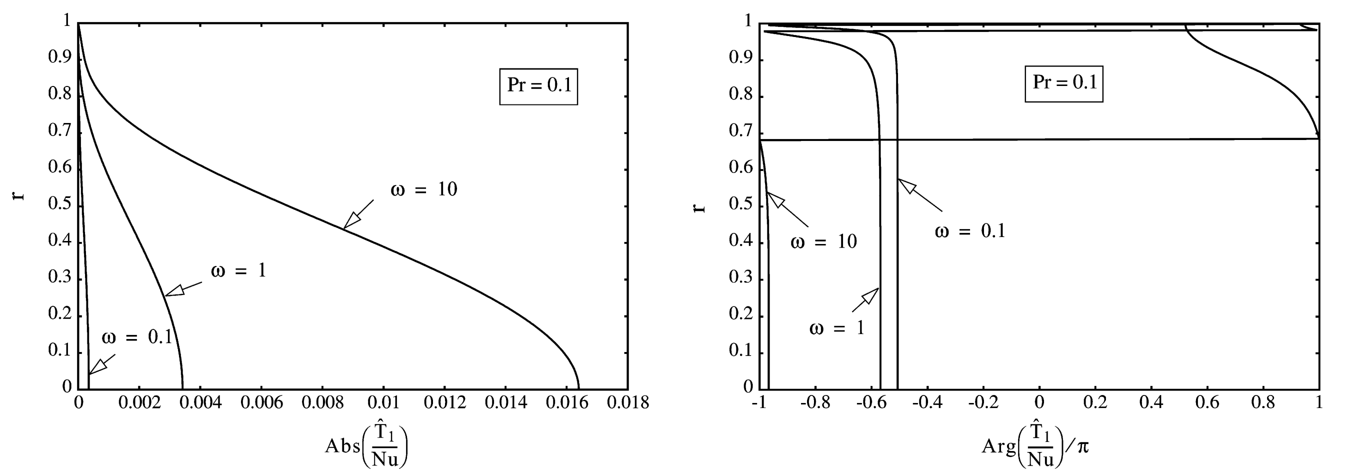
as the value of the Prandtl number increases. The plot for the value $Pr = 1$ was produced numerically by taking $Pr = 0.999$ or 1.001 before we did the analysis leading to Eq. (40).

One general remark can be made. It is obvious from Eq. (19) that if ωPr is large compared with unity then the time-derivative term in the energy equation dominates over the conduction term. Since ωPr appears multiplied by $\hat{T}_1$, it follows that $\hat{T}_1$ becomes small, and so Nu_1 becomes small, when Pr becomes large for large or intermediate values of ω.

4.2. Circular tube

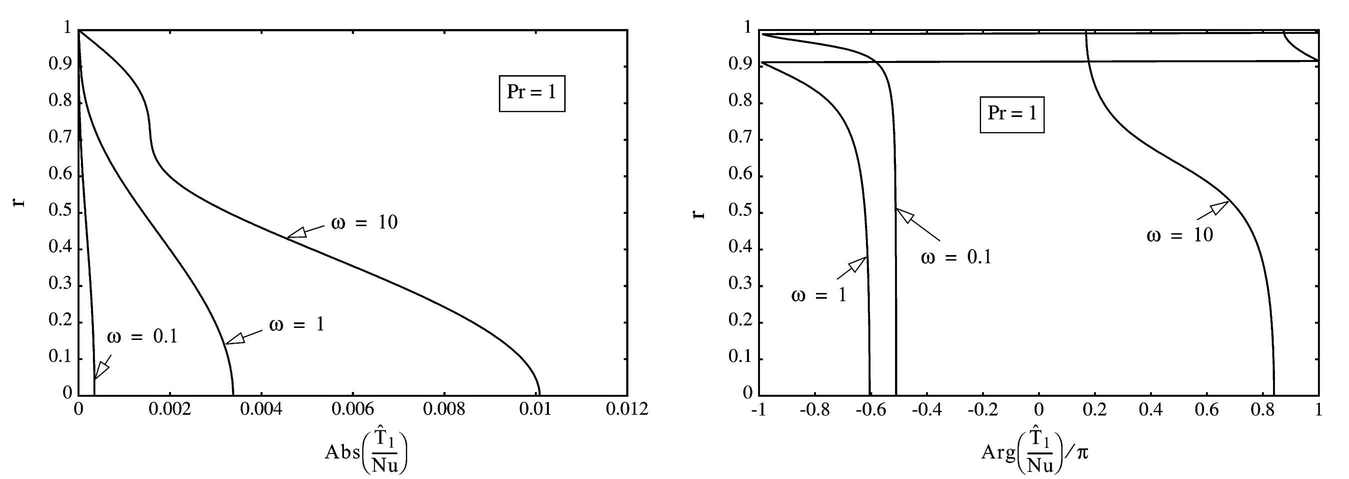
The corresponding results for the velocity and temperature profiles for the circular tube are presented in Figs. 7–11. These show that the results for the circular tube are qualitatively similar to those for the parallel-plates channel, as one would expect.

The results for the Nusselt number are presented in Fig. 12. Clearly they are qualitatively similar to those for the parallel



(a) (b)

Fig. 9. Plot of the profile of (a) the modulus, (b) the argument of the normalized perturbation temperature, for various values of the dimensionless frequency, for the circular tube. Prandtl number $Pr = 0.1$.



(a) (b)

Fig. 10. As for Fig. 9, but with $Pr = 1$.

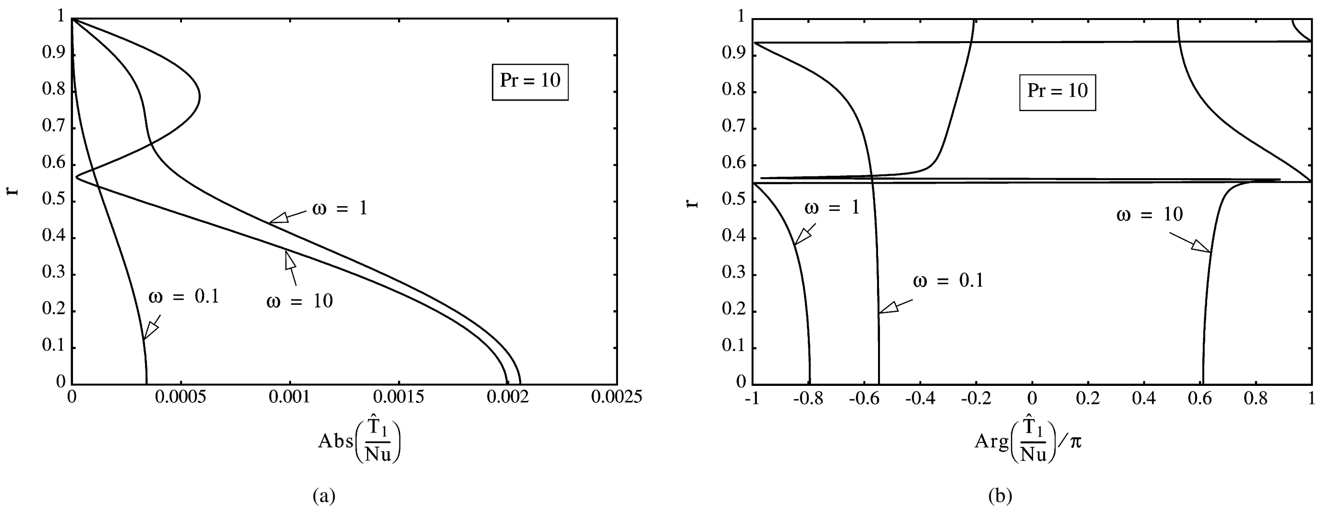


Fig. 11. As for Fig. 9, but with $Pr = 10$.

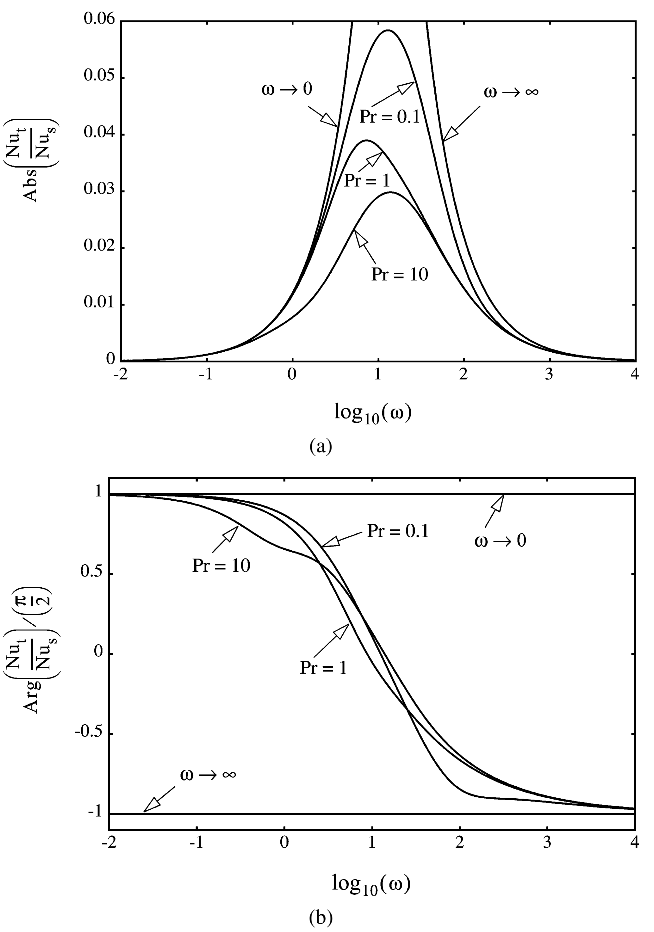


Fig. 12. Plots of the (a) the modulus, and (b) the argument as a fraction of $\pi/2$, of Nu_t/Nu_s, where $Nu = Nu_s + Nu_t\,\varepsilon e^{i\omega t}$, for various values of the Prandtl number Pr, for the case of the circular tube. Here $Nu_s = 48/11$.

plate channel. For the circular tube the peaks in the magnitude of Nu_t/Nu_s are higher by a factor of about 2.5, and they occur at values of ω that are larger by a factor of order 3, in comparison with the values for the parallel-plates channel.

5. Conclusions

Using a perturbation approach, we have obtained analytical expressions for the velocity, temperature distribution, and transient Nusselt number for the problem of forced convection, in a parallel-plates channel or a circular tube, produced by an applied pressure gradient that fluctuates with small amplitude harmonically in time about a non-zero mean.

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